

## Atom-photon entanglement generation and distribution

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We extend an earlier model by Law and Kimble [J. Mod. Opt. **44**, 2067 (1997)] for a cavity QED based single-photon-gun to atom-photon entanglement generation and distribution. We illuminate the importance of a small critical atom number on the fidelity of the proposed operation in the strong-coupling limit. Our result points to a promisingly high purity and efficiency using currently available cavity QED parameters, and sheds new light on constructing quantum computing and communication devices with trapped atoms, and high- $Q$  optical cavities.

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Entanglement lies at the heart of quantum information and computing science [1,2]—it is responsible both for the mysteries of quantum cryptography [3] and teleportation [4] as well as for the exponential speed-up promised by Shor’s factoring algorithm [5]. It is widely believed that the progress of quantum information science, on the experimental front, will track closely the progress of entanglement generation in the laboratory. Until recently [6,7], most experimental realizations of entanglement came almost exclusively from photon down-conversion using nonlinear crystals where a single pump photon spontaneously converts into two correlated photons satisfying energy and momentum conservation [8,9]. Although the individual polarization states of photons are easily controlled, and their quantum coherence can be preserved over many kilometers of an optical fiber [10], photons cannot be stored for long, and manipulations of collective entangled state present considerable difficulties even when photons are confined inside the same cavity.

The creation of long-lived entangled pairs with material particles (atoms and ions), on the other hand, is a relatively recent pursuit [11–13], spurred on in large part by developments in quantum logic and computing. These experimental efforts have been very successful and are highlighted by the demonstration of a four-ion entangled state [14], using a proposal with trapped ions due to Molmer and Sorensen [15]. However, the scaling of this technology to larger numbers of qubits ( $>10$ ) is less certain, and it is unlikely that quantum information stored exclusively in material particles will ever be effectively distributed to remote locations as required for most quantum communication and distributed computing applications.

Given the current state of affairs, there is a pressing need for systems capable of integrating the relative strengths of material particle-based entanglement and photon-based entanglement, wherein the former provides reliable quantum information storage and local entanglement capabilities, the latter provides quantum communication capabilities over long distances. It is important to develop capabilities for reliably converting and transferring quantum information between material and photonic qubits.

In this paper, we develop a system composed of a single trapped atom inside a high- $Q$  optical cavity for deterministic generation of atom-photon entanglement and its subsequent

distribution via the well-directed photon from a high- $Q$  optical cavity. For a large class of quantum communication protocols (including cryptography protocols, teleportation, entanglement purification, etc.), one always begins with the following statement: “Imagine that Alice has an entangled pair of particles, and she sends one particle to Bob...” Our proposed system, if implemented properly, will supply such a device.

The physical model for our system represents a direct extension of an earlier proposal by Law and Kimble [16] for a deterministic single-photon (or “Fock states”) source [17], an indispensable device for some quantum cryptographic applications [3]. Although significant progress has already been made along the direction of a deterministic single-photon source [17], to date, most experiments still rely on an attenuated laser pulse for a single photon.

In the system studied by Law and Kimble [16], a single atom is placed inside a high- $Q$  optical cavity. A pictorial illustration of the required energy-level structure is reproduced in the left panel of Fig. 1, where a three-level  $\Lambda$ -type atom with one excited state  $|e\rangle$  and two long-lived ground states ( $|g_1\rangle$  and  $|g_2\rangle$ ) are coupled, respectively, to a classical pump field (in solid line) and the cavity field (in dashed line). In the strong-coupling limit  $g \gg \gamma$  and  $g \gg \kappa$  [18], the dominant absorption-emission process consists of an atom starting in  $|g_1\rangle$ , pumped into the excited state  $|e\rangle$ , which then decays via the cavity into  $|g_2\rangle$  [16,19]. Following the emission, an external laser field driving the transition  $|g_2\rangle \rightarrow |e\rangle$  can reset the atom to state  $|g_1\rangle$  and prepare it for the next photon emission. Such a single-photon gun is expected to reach a rep-rate  $\sim \kappa$ , which is typically several megahertz. An alternative approach based on adiabatic passage for a deterministic or “push-button” single-photon source was considered in Ref. [19]. In the above,  $g = dE_0/\hbar$  is the dipole coupling between the atom and a single cavity photon (of angular frequency  $\omega$ ) field  $E_0 = \sqrt{2\pi\hbar\omega/V}$  confined in a mode volume  $V$ .  $d$  is the electric dipole matrix element and  $\gamma(\kappa)$  is the excited atom (cavity) decay rate.

A simple extension of the three-level atom to a four-level one as illustrated in the right panel of Fig. 1 consists of our model. The excited state  $|e\rangle$  is now resonantly coupled to

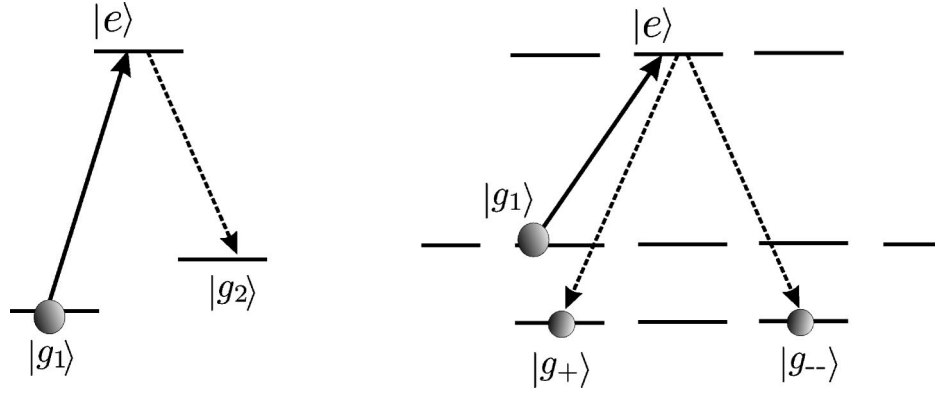


FIG. 1. Atom-photon entanglement illustrations.

both states  $|g_+\rangle$  and  $|g_-\rangle$  through the left and right circular polarized cavity photon fields [20]. Following Law and Kimble [16], the coherent part (in the rotating-wave approximation) of our model Hamiltonian can be expressed as

$$H_0 = \hbar g(a_L \sigma_{e,g_-} + a_L^\dagger \sigma_{g_-,e} + a_R \sigma_{e,g_+} + a_R^\dagger \sigma_{g_+,e}) + \frac{1}{2} \hbar \Omega(t) (\sigma_{g_1,e} + \sigma_{e,g_1}), \quad (1)$$

where  $\sigma_{\mu,\nu}(t=0) = |\mu\rangle\langle\nu|$  ( $\mu, \nu = g_1, e, g_-, g_+$ ) are atomic projection operators.  $a_\xi$  and  $a_\xi^\dagger$  ( $\xi = L, R$ ) are annihilation and creation operators for the quantized cavity field.  $\Omega(t)$  denotes the coupling between the atom and the external classical field. Including the non-Hermitian dynamics due to both atomic spontaneous decays and the cavity decay, the master equation of our system becomes

$$\frac{d}{dt} \rho = -\frac{i}{\hbar} [H_0, \rho] + \kappa \sum_{\xi=L,R} (2a_\xi \rho a_\xi^\dagger - a_\xi^\dagger a_\xi \rho - \rho a_\xi^\dagger a_\xi) + \sum_{\mu=g_1, g_-, g_+} \frac{\gamma \beta_\mu}{2} (2\sigma_{\mu,e} \rho \sigma_{e,\mu} - \sigma_{e,e} \rho - \rho \sigma_{e,e}), \quad (2)$$

where  $\beta_\mu$  denotes the branching ratio of the atomic decay to levels  $|\mu\rangle$  and  $\beta_{g_1} + \beta_{g_-} + \beta_{g_+} = 1$ . Similar to the Law protocol [16], the system is prepared in state  $|g_1\rangle$  with no photon in the cavity, after the classical field  $\Omega(t)$  is tuned on for a period  $T_0$ , a single photon (with either a left or a right circular polarization) is generated in the cavity, which immediately transmits outside the cavity in the bad cavity limit of

$$\kappa \gg g^2/\kappa \gg \gamma. \quad (3)$$

The probability of detecting a photon with a given polarization is easily computed according to

$$P_\xi(t) = 2\kappa \int_0^t \langle a_\xi^\dagger(t') a_\xi(t') \rangle dt'. \quad (4)$$

To gain more physical insight, we describe the dynamic evolution of the system using the non-Hermitian effective Hamiltonian [21]

$$H_{\text{eff}} = H_0 - i\hbar \kappa (a_L^\dagger a_L + a_R^\dagger a_R) - i\hbar \frac{\gamma}{2} \sigma_{e,e}. \quad (5)$$

Using the combined basis of atomic internal state ( $\mu = g_1, e, g_-, g_+$ ) and the Fock basis of cavity photons  $|\mu, n_L, n_R\rangle$ , we can analyze the dynamics of the photon-emission process. Limited to 0 and 1 photon numbers  $n_{L/R} = a_\xi^\dagger a_\xi$ , the pure state wave function from Eq. (5),

$$|\psi(t)\rangle = a_{g_1} |g_1, 0, 0\rangle + a_e |e, 0, 0\rangle + a_{g_-} |g_-, 1, 0\rangle + a_{g_+} |g_+, 0, 1\rangle, \quad (6)$$

obeys the conditional dynamics described by  $i\hbar |\dot{\psi}\rangle = H_{\text{eff}} |\psi\rangle$ . Explicitly, we find

$$\begin{aligned} i\dot{a}_{g_1}(t) &= \frac{1}{2} \Omega(t) a_e, \\ i\dot{a}_e(t) &= \frac{1}{2} \Omega(t) a_{g_1} + g a_{g_-} + g a_{g_+} - i\frac{\gamma}{2} a_e, \\ i\dot{a}_{g_-}(t) &= g a_e - i\kappa a_{g_-}, \\ i\dot{a}_{g_+}(t) &= g a_e - i\kappa a_{g_+}. \end{aligned} \quad (7)$$

When the classical pump field satisfies the condition of  $\Omega(t) \ll g^2/\kappa$ , the approximation solution to Eq. (7) becomes

$$\begin{aligned} a_{g_1}(t) &\approx \exp\left[-\frac{1}{2(4g^2/\kappa + \gamma)} \int_0^t \Omega^2(t') dt'\right], \\ a_e(t) &\approx -i \frac{\Omega(t)}{4g^2/\kappa + \gamma} a_{g_1}(t), \end{aligned} \quad (8)$$

$$a_{g_\pm}(t) \approx -i \frac{g}{\kappa} a_e(t),$$

given the initial condition of  $a_{g_1}(0) = 1$  and  $a_{e/g_-/g_+}(0) = 0$ . Clearly, the left and right polarized modes are equally populated if their couplings to the cavity mode are identical. The conditional state of the system then becomes [22,23]

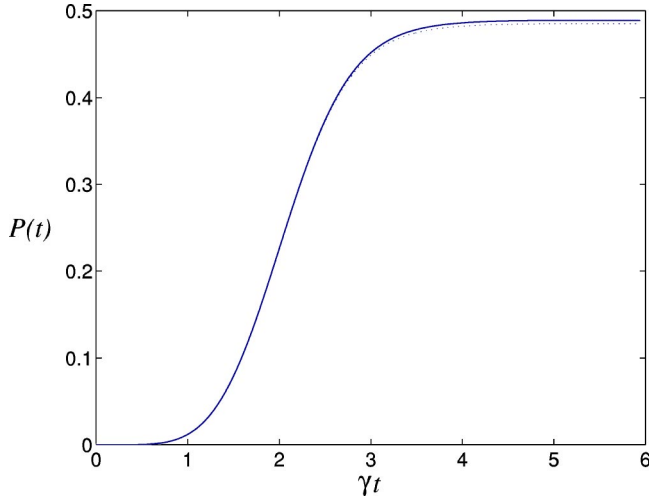


FIG. 2. The conditional probability for the emission of a cavity photon.

$$\frac{1}{\sqrt{2}}(|g_{-}\rangle|n_L=1,0\rangle + |g_{+}\rangle|0,n_R=1\rangle), \quad (9)$$

an atom-photon entangled state.

We have performed detailed numerical simulations for a classical pump field of the form

$$\Omega(t) = \Omega_0 \sin^2\left(\frac{\pi t}{T_0}\right), \quad 0 \leq t \leq T_0 \quad (10)$$

with  $(g, \kappa, \gamma, \Omega_0) = (2\pi)(45, 45, 4.5, 45)$  MHz and  $T_0 = 6/\gamma = 210$  ns. We present selected results in Figs. 2 and 3. Clearly, our model works in exactly the same way as the original Law protocol [16]. The only difference being now that the confirmed emission of a cavity photon is accompanied with atom-photon entanglement. With the above parameters, we find that the conditional probability for a left or right polarization photon rapidly increases to about 49%. For comparison, we have solved both the conditional wave-function dynamics as well as the complete master

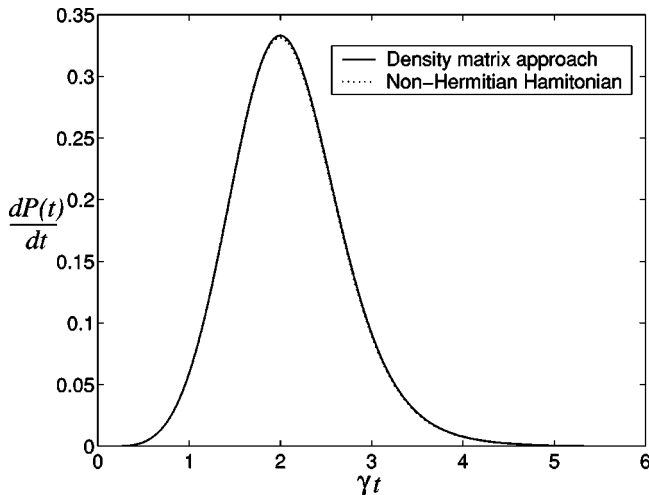


FIG. 3. Cavity emission rate.

equation dynamics. The non-Hermitian Hamiltonian dynamics gives a slightly lower value for final photon emission probability because it excludes repeated spontaneous decays. When the atom in  $|e\rangle$  decays to  $|g_1\rangle$ , it may be reexcited by the classical field before emitting the photon into the right cavity mode. Such an event should be excluded in order to have a final state with atom cavity coherence, yet it is included in the master equation solution.

We note that the bad cavity limit, or the operating condition as specified by Eq. (3), in fact corresponds to the cavity QED system not in the strong-coupling limit. Thus the cavity photon decays immediately once created, and this allows for an adiabatic description by eliminating the atomic dynamics in the cavity. The condition of  $g^2 \gg \kappa\gamma$  is in fact the same requirement of a large cooperativity parameter ( $C \propto g^2/\kappa\gamma$ ) or a small critical atom number ( $n_c \propto \kappa\gamma/g^2$ ) as in the strong-coupling limit. It turns out that this parameter is an important characterization for the fidelity of several important quantum computing protocols of atomic qubits inside high- $Q$  cavities [24]. We now further illuminate this in terms of the basic element of quantum information exchange between a cavity and an atomic qubit.

We consider a three-level  $\Lambda$ -type coupling scheme as in the left panel of Fig. 1. When the classical field  $\Omega(t)$  is Raman resonant with respect to the cavity photon (assuming a perfect compensation for ac Stark shifts [25]), while strongly off-resonant with respect to the atomic transition  $|e\rangle \leftrightarrow |g_2\rangle$ , the two states  $|g_1, 0\rangle$  and  $|g_2, 1\rangle$  are effectively coupled directly through a Rabi frequency  $\Omega_{\text{eff}}$  and an effective atomic decay rate  $\gamma_{\text{eff}}$  given by

$$\Omega_{\text{eff}} = \frac{1}{2} \frac{\Omega g}{\Delta}, \quad (11)$$

$$\gamma_{\text{eff}} = \frac{1}{4} \frac{\Omega^2}{\Delta^2} \gamma, \quad (12)$$

where  $\Delta = \omega_L - (\omega_e - \omega_1)$  is the pump field detuning.

To expect coherent dynamics for state mapping [26] according to

$$(\alpha|g_1\rangle + \beta|g_2\rangle) \otimes |0\rangle \rightarrow |g_2\rangle \otimes (\alpha|1\rangle + \beta|0\rangle), \quad (13)$$

one requires  $\Omega_{\text{eff}} \gg \gamma_{\text{eff}}$  and  $\Omega_{\text{eff}} \gg \kappa$ , which reduces to

$$\frac{\kappa}{g} \ll \frac{\Omega}{\Delta} \ll \frac{g}{\gamma}, \quad (14)$$

thus  $g/\gamma \gg \kappa/g$  or  $g^2 \gg \kappa\gamma$  [27].

We have performed extensive numerical simulations to check this understanding. First for the Raman scheme and a constant  $\Omega$ , we define *success rate* (Fig. 4) [16] as the conditional probability for an atom to end up in state  $|g_2\rangle$  (from initially in state  $|g_1\rangle$ ), i.e., conditioned on the system to experience no spontaneous emission from either the atom or the cavity. The numerical results are given in Fig. 4, which shows a weak dependence on the classical field detuning  $\Delta$ . Typically, we find that the optimal condition corresponds to  $\Omega/\Delta \sim 0.2-0.75$ .

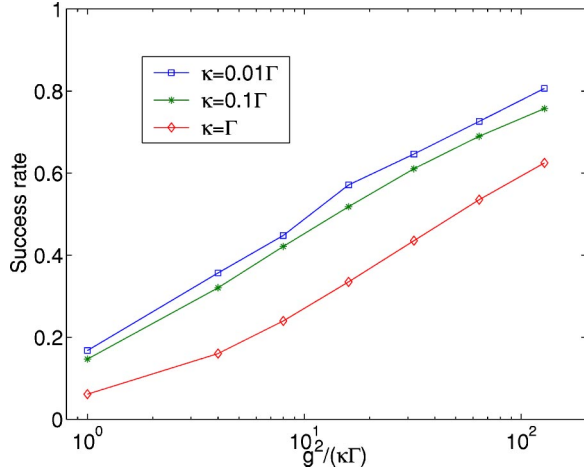


FIG. 4. The success rate of mapping the atomic state  $\alpha|g_1\rangle + \beta|g_2\rangle$  into the cavity state  $\alpha|1\rangle_C + \beta|0\rangle_C$ . The worst case scenario of  $\alpha=1$  and  $\beta=0$  is considered here. States with nonzero  $\beta$ 's generally lead to proportionally larger success rates. We have used  $\gamma = (2\pi)20$  MHz and  $\Delta = 50\gamma$ .

The second figure of merit applies to the operation of the atom + cavity system as a photon gun, the aim of our proposed model. In this case, the probability of emission, or the *emission rate* into the cavity mode is used. In a sense, it measures the photon-gun quality. The results from our numerical survey are illustrated in Fig. 5. It is interesting to note that the results  $Do$  depend on the detuning, essentially reflecting an unbalanced choice of  $\kappa$  with  $\gamma_{\text{eff}}$ . We also note that together with the probability of the atomic spontaneous emission, the two add to unity in the long-time limit.

Building on several current experiments, it seems possible to achieve  $g^2/(\gamma\kappa) = 30$  [28–30], a condition for very efficient photon gun according to our calculations; in the strong-coupling limit, this also becomes a promising parameter regime for converting an atomic qubit into a flying qubit (of 0 and 1 photons).

For comparison, we have also compared the state mapping, Eq. (13), with the counterintuitive pulse sequence for

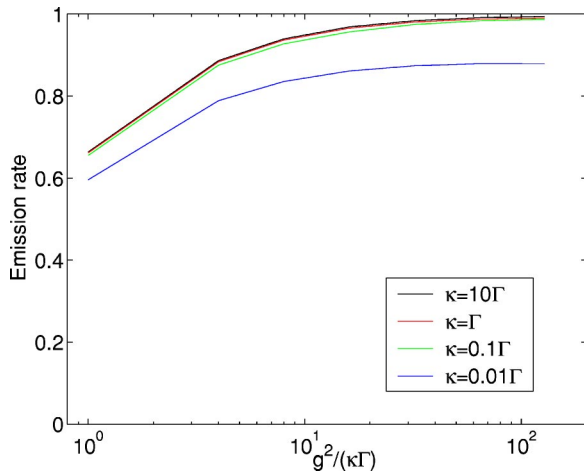


FIG. 5. The optimal photon-gun quality for  $\gamma = (2\pi)20$  MHz and  $\Delta = 50\gamma$ .

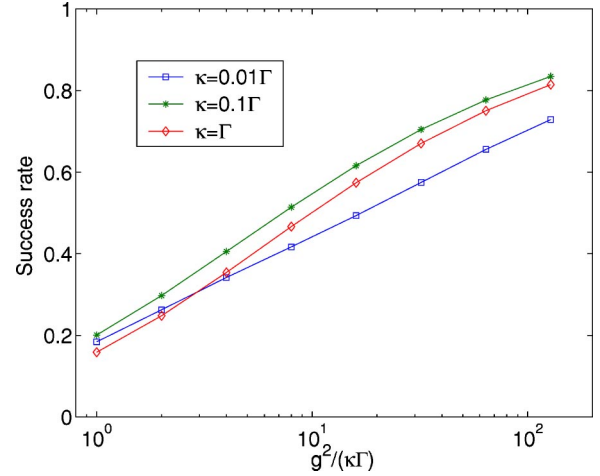


FIG. 6. The same as in Fig. 4, but with the adiabatic passage protocol. The results are optimized and are observed to be less sensitive on the ratio of  $\kappa/\gamma$ .  $\gamma = (2\pi)20$  MHz.

adiabatic passage [26,31–33]. The best numerical results are shown in Figs. 6 and 7, respectively. We note that in this case the numerical survey is rather cumbersome as we are looking at a three-dimensional ( $\Omega$ ,  $\Delta$ , and  $\delta$ ) optimization search for each data point. The Raman differential detuning is defined as  $\delta = (\omega_L - \omega_C) - (\omega_2 - \omega_1)$ .

Finally, we briefly comment on the effect of atomic motion on the discussed protocol for atom-photon entanglement. In all current optical cavity QED systems, the coupling strength  $g(\vec{r})$  between the quantum cavity field and the atom, is position dependent, with  $\vec{r}$  being the atomic center of mass coordinate. The variation of  $g(\vec{r})$  due to the standing wave cavity mode along the cavity axis leads to entanglement between the atomic motion and its internal state, which in the extreme case can cause a complete loss of coherence/entanglement between the atom and the emitted photon, if the atomic center of mass wave packet is delocalized to a size ( $\delta r$ ) comparable to or larger than the cavity mode wavelength  $\lambda_c$ . Typically one requires the so-called Lamb-Dicke limit, or  $\delta r \ll \lambda_c$ , to enforce an approximately constant  $g(\vec{r})$

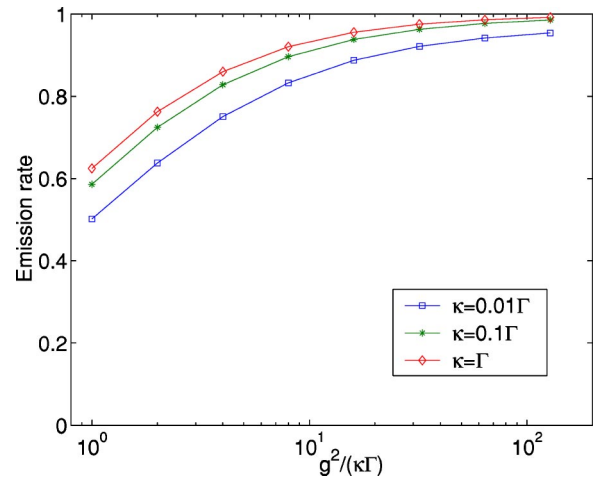


FIG. 7. The same as in Fig. 5, but with the adiabatic passage protocol. The results are optimized and for  $\gamma = (2\pi)20$  MHz.



over the whole atom. The small sized wave packet can be prepared by cooling atomic motion to the ground state of an external harmonic trap as for trapped ions. In this limit, effects of atomic recoil become negligible as the recoil energy is much less than the trap excitation quanta. The dependence of the fidelity for the state transfer protocol, Eq. (13), on the Lamb-Dicke parameter  $\eta_c = 2\pi\delta r/\lambda_c$  has already been extensively investigated before with numerical simulations [34]. For the application of the recently suggested motional insensitive dark state protocol [35] to state transfer, Eq. (13), nonadiabatic motional effect has also been studied in great detail [36].

In summary, we have proposed a simple and efficient implementation for a deterministic generation of atom-photon entanglement. Our arrangement can be directly

adopted for distributing entanglement shared between any two parties as the single photon can be propagated to reach a distant party. Successful realization of the controlled interactions between a single trapped atom and a cavity photon will represent an important milestone in quantum information physics.

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