Editors' Suggestion

Long-Lived Squeezed Ground States in a Quantum Spin Ensemble

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We generate spin squeezed ground states in an atomic spin-1 Bose-Einstein condensate tuned near the quantum-critical point separating the different spin phases of the interacting ensemble using a novel nonadiabatic technique. In contrast to typical nonequilibrium methods for preparing atomic squeezed states by quenching through a quantum phase transition, squeezed ground states are time stationary with a constant quadrature squeezing angle. A squeezed ground state with 6–8 dB of squeezing and a constant squeezing angle is demonstrated. The long-term evolution of the squeezed ground state is measured and shows gradual decrease in the degree of squeezing over 2 s that is well modeled by a slow tuning of the Hamiltonian due to the loss of atomic density. Interestingly, modeling the gradual decrease does not require additional spin decoherence models despite a loss of 75% of the atoms.

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Exploiting quantum entanglement to surpass the standard quantum limit of measurement precision is an active frontier of research motivated by practical applications, such as improving the sensitivity of atomic clocks and atom and optical interferometers [1–3]. An important class of entangled states providing metrological gain are spin squeezed states of atomic ensembles, which have a reduced or squeezed variance of a collective spin component compared to an uncorrelated ensemble [4,5]. Atomic spin squeezing has been created using a variety of methods including quantum nondemolition projective measurements [6–11], atom-cavity interactions [12,13], interacting trapped ions [14,15], and in Bose-Einstein condensates (BECs) [16–22].

BECs are bosonic many-body systems often restricted to occupy a small number of modes (e.g., external trapping sites [16,17] or internal hyperfine states [18–22]) that can be described using collective spin states. Such condensates are governed by simple Hamiltonians that have different quantum ground state phases depending on tunable parameters. Most of the experimental demonstrations of spin squeezing in condensates have utilized nonequilibrium evolution following a deep quench across a quantum phase transition [21,23–25] or parametric or Floquet excitation [26,27]; both methods start from an initially uncorrelated state and create the entanglement dynamically in a highly excited state.

On the other hand, there has been long-standing interest in the nature of entanglement of the different ground state phases of these systems [28–32]. Entangled ground states are central to adiabatic quantum computing and understanding strongly correlated many-body systems, and there are also compelling applications to quantum enhanced metrology. To this last point, there have been experiments using adiabatic [33] or quasiadiabatic [34,35] evolution to create Dicke states, twin-Fock states [36], and pseudo-spin-1/2 number squeezed states [17,37,38].

The focus of this Letter is the creation and investigation of spin squeezed ground states that occur near the quantum-critical point (QCP) separating the different phases of the Hamiltonian. The properties of the squeezed ground state are determined by the properties of the final Hamiltonian rather than the details of the nonequilibrium evolution and are thus easier to characterize and control. In particular, although achieving the squeezed ground state requires careful Hamiltonian tuning, the minimum squeezed quadrature angle of the ground state has a fixed orientation independent of the final Hamiltonian parameters such as density and magnetic field. In contrast, for squeezed states created from nonequilibrium quench dynamics, the minimum squeezing quadrature angle is both time and atom number dependent [21]. Additionally, spin squeezed ground states provide opportunities to more carefully investigate long-term evolution of entanglement in spin ensembles because the squeezing is now in a stationary state. Toward this end, we demonstrate a long-lived squeezed ground state with 6–8 dB of squeezing that maintains a constant quadrature orientation. The degree of squeezing gradually decreases over 2 s due to atom loss. This decrease is well modeled by a slow tuning of the Hamiltonian and does not require additional spin decoherence models despite a loss of 75% of the atoms.

A distinguishing feature of the investigation described in this Letter is the use of a novel, nonadiabatic double-quench method [39] that significantly shortens the state preparation time compared to adiabatic methods typically used to approach the QCP. This improves both the fidelity of the target state and limits loss of entanglement due to uncorrelated atom losses. Our method is related to a shortcut to adiabaticity and other optimal control techniques [40] that have been widely considered theoretically for fast production of highly entangled states [41–46]. Experimental demonstrations of these types of control techniques have been limited to motional control of ultracold atoms [47–49] with the exception of an experiment demonstrating the creation of spin entanglement using trapped ions [50].

Our experiment uses small spin-1 atomic rubidium condensates confined in an optical trap. The condensate spin dynamics are described by the Hamiltonian [21]

$$\hat{H} = \frac{c}{2N}\hat{S}^2 - \frac{q}{2}\hat{Q}_z, \tag{1}$$

where $\hat{S}$ is the collective spin operator, and $\hat{Q}_z$ is a collective nematic or quadrupole operator. The coefficient $c/2N$ is the collisional spin interaction energy per particle, and $q \propto B^2$ is the quadratic Zeeman energy per particle for a magnetic field oriented in the z direction. For the ^{87}Rb $F = 1$ hyperfine state, $c < 0$ and the condensate has a ferromagnetic (FM) phase and a polar phase separated by a QCP at $q = 2|c| \equiv q_c$ (see Sec. I in the Supplemental Material [51]).

We begin by describing the basic idea behind the experiment. The starting point is a spin-1 condensate prepared in the $m_F = 0$ Zeeman state at a high magnetic field such that $q = q_0 \gg q_c$, and the spin interaction term of the Hamiltonian can be ignored. This is an uncorrelated ground state with Heisenberg uncertainty for the complimentary observables $\Delta S_x \Delta Q_{yz} = N$, where $\widehat{S_x}$ is the collective spin operator in the x direction, and $\hat{Q}_{yz}$ is the collective nematic operator between the y and z directions. Throughout the text, operators are indicated by carets, while the corresponding symbol without the caret indicates their expectation value.

The phase space of the system can be visualized on a Bloch sphere of $\{S_x, Q_{yz}, Q_z\}$ (see Fig. 1) where the ground state is located at the $Q_z = 1$ pole with symmetric uncertainties in S_x and Q_{yz}. In earlier demonstrations of spin-nematic squeezing [19,21], the squeezing was generated by nonequilibrium evolution from an unstable fixed point following a deep quench across the QCP to the FM phase as shown in Fig. 1(i). In this Letter, we are interested in creating squeezing in the polar phase in the neighborhood of the QCP and, in particular, creating squeezing in the ground state of the system with $q \gtrsim q_c$. We again begin with a sudden quench from q_0, but now to a final field above the QCP, $q_i \gtrsim q_c$. At this field, the ground state remains polar in character, but the spin interactions are no longer negligible and distort the semiclassical orbits of the system into ellipses. Subsequent evolution of the initially symmetric uncertainties gives rise to periodic squeezing [60] and unsqueezing with a frequency $\omega_i = \sqrt{q_i(q_i - q_c)}$ as shown in Figs. 1(b)–1(f) from the energy gap [26]. Of course, this is an excited state of the system with dynamically evolving observables, in this case the uncertainties ΔS_x and ΔQ_{yz}. Although this state is not a ground state of the Hamiltonian $\hat{H}(q_i)$, it is the ground state of another Hamiltonian $\hat{H}(q_f)$ where $q_i > q_f > q_c$. To end with the condensate in a ground state, we perform a second quench with a timing and final field value chosen to match the evolving state with the shape of the ground state of the final Hamiltonian. This second quench results in the system in the ground state of $\hat{H}(q_f)$ as shown in Figs. 1(g) and 1(h).

The ground state of $\hat{H}(q_f)$ exhibits squeezing in the variance of Q_{yz} by an amount [39]

$$\xi^2_{Q_{yz}} = \Delta Q^2_{yz}/N = 1/\eta, \tag{2}$$

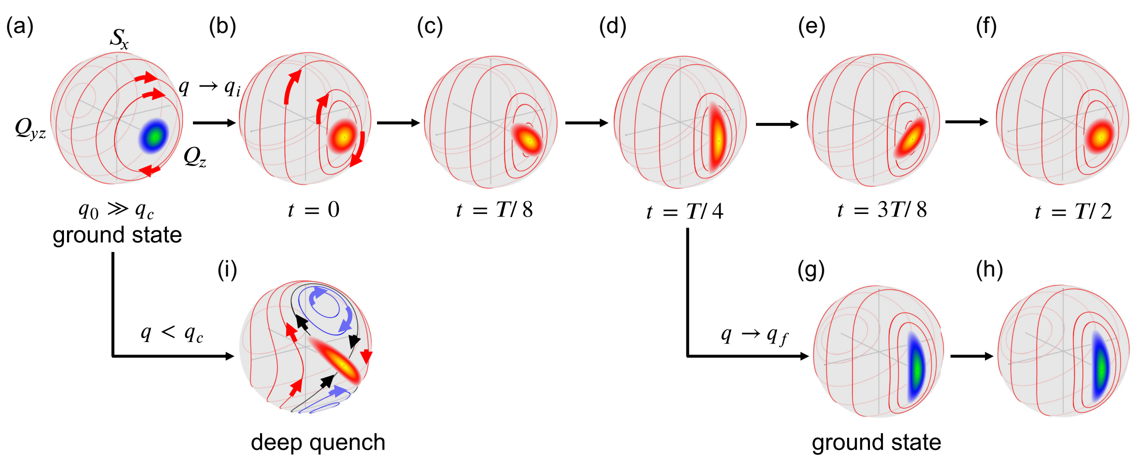

FIG. 1. The spin-1 states in the $\hat{S}_z = 0$ subspace and their evolution can be visualized on a $\{S_x, Q_{yz}, Q_z\}$ Bloch sphere. (a) The initial state is an uncorrelated ground state at $q \gg q_c$ with symmetric uncertainties in S_x and Q_{yz}. (b)–(f) Following a sudden quench to $q_i \gtrsim q_c$ at $t = 0$, the ground state remains polar, but the fluctuations evolve periodically along elliptical orbits with a frequency $\omega_i = 2\pi/T$. (g), (h) A second quench at $T/4$ to a suitably chosen q_f will deexcite the condensate into a stationary squeezed ground state. (i) Standard nonequilibrium method of generating spin-1 squeezing following a sudden deep quench across the QCP to the FM phase [19,21].

where $1/\eta = \sqrt{1 - q_c/q_f}$, and antisqueezing by an amount η in the complementary observable S_x. In order to end in the ground state, the second quench needs to occur at a time $T/4 = \pi/(2\omega_i)$, and q_f needs to satisfy the relation $(q_i - q_c)/q_i = 1/\eta$. Of course, it is also possible to adiabatically ramp the Hamiltonian directly from $q_0 \to q_f$, but the double-quench shortcut method is at least $\sqrt{\eta}$ faster than the shortest adiabatic ramp time $T_{\mathrm{adiab}} \geq 2\pi\eta/q_f$ (see Ref. [39] for details).

We now turn to the experimental measurements. We first investigate the single quench nonequilibrium periodic squeezing following Figs. 1(b)–1(f). A condensate of 50 000 atoms is prepared in the $m_F = 0$ state in an optical dipole cross trap at a high field, $q_0 = 5q_c$. Following a sudden quench to q_i, the condensate is allowed to freely evolve. The mean spin populations do not significantly change as the condensate is still in the polar phase; however, the spin fluctuations do evolve. In Fig. 2(a), measurements of the time evolution of ΔQ_{yz} are shown that exhibit periodic squeezing and unsqueezing; measurements of ΔS_x show complimentary behavior of periodic antisqueezing (see Sec. III in [51]). In Fig. 2(b), tomographic measurements of the fluctuations at the point of maximum Q_{yz} squeezing ($t = T/4$) are shown. Each data point corresponds to a measurement at a different quadrature phase $\theta = \theta_s/2$, where θ_s is the relative phase between $m_F = 0$ and $m_F = \pm 1$ spin components:

$$\xi_\theta^2 = \Delta(S_x \cos\theta + Q_{yz} \sin\theta)^2/N. \tag{3}$$

The data show up to −6 dB of squeezing and symmetric antisqueezing. The data are compared with simulations that show good qualitative agreement; however, it is necessary to scale the simulations by $\xi^2 = (\xi^2_{\mathrm{sim}})^{0.7}$ to quantitatively match the observed squeezing; a possible explanation is that normally negligible effects such as magnetic anisotropy [61] or dipolar interactions [62] become significant near the critical point where the energy scale goes to zero. In the figures throughout, the simulations are scaled to account for this discrepancy.

Also shown in Fig. 2 are data taken following the double-quench sequence $q_0 \to q_i \to q_f$ designed to achieve the squeezed ground state of $\hat{H}(q_f)$. In Fig. 2(a), the data show that following the second quench to q_f, the time evolution of ΔQ_{yz} remains constant at the level of the maximum squeezing previously observed, as expected for the ground state. The data are compared with a simulation result including a ± 0.1 Hz uncertainty in c (see Sec. II in [51]). The precise values of T and q_f are determined from the single quench data. Tomographic measurements of the fluctuations of the ground state shown in Fig. 2(b) taken at a much later time ($t \sim 3T/4$) are indistinguishable from measurements made of the periodic squeezing at ($t = T/4$), as expected. Furthermore, in addition to a constant squeezing amplitude, the maximum squeezing angle (the minimum quadrature angle) $\theta_{s,\mathrm{min}} = \min\{\xi^2_\theta|\theta_s\} = -\pi$ remains constant following the second quench. This is in stark contrast to the deep quench method [Fig. 1(i)] for which $\theta_{s,\mathrm{min}}$ is a function of c, q and evolves dynamically (see Sec. I in [51]). The experimental data are corrected for the photon shot noise and the background imaging noise and the detection limit of the squeezing is −7 dB (see Sec. II in [51]). From the measurement of −6 dB of squeezing, it is possible to determine the entanglement breadth of the spin ensemble [22,35,63]. From this, we can conclude that a nonseparable (entangled) subset of 600 particles is detected in the squeezed ground state (see Sec. III in [51]). For comparison, we have also used an adiabatic ramp method to

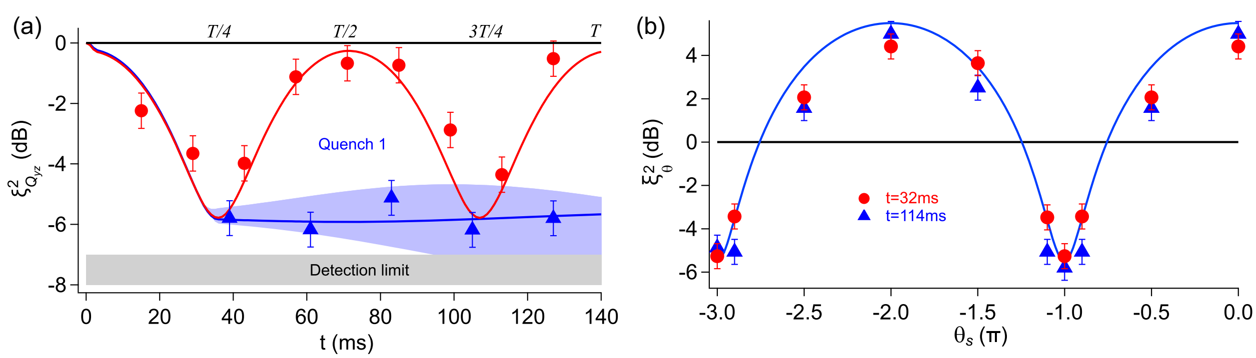

FIG. 2. Time-stationary squeezing and periodic squeezing. (a) Measurement of time-stationary squeezing in the ΔQ_{yz} observable following the double-quench sequence $q_0 \to q_i \to q_f$ designed to create a squeezed ground state at q_f (blue triangles). Quench 1 is defined as $q_i = 1.16q_c$ and $q_f = 1.04q_c$. These data are compared to a single quench $q_0 \to q_i$ (red circles), which exhibit periodic squeezing and unsqueezing in ΔQ_{yz}. Simulation results with $c = -8.2 \pm 0.1$ Hz (blue shaded area) are compared with the data. All simulations are scaled $\xi^2 = (\xi^2_{\mathrm{sim}})^{0.7}$ to match the observed squeezing. (b) Tomographic measurements of the fluctuations at $t = T/4$ (red circles) and at a much later time ($t \sim 3T/4$) after the second quench (blue triangles). The error bars indicate the standard deviation of measured variance determined from 100 repeated measurements per data point.

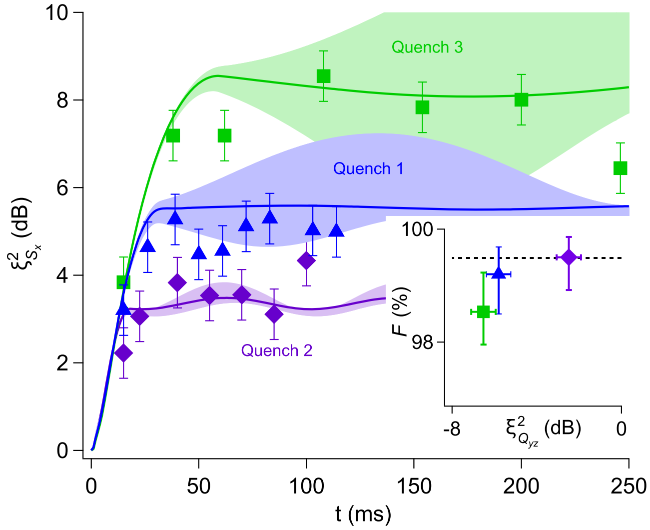

FIG. 3. Measurement of $\xi^2_{S_x}$ versus t following the double-quench sequence for different q_f. The solid lines are simulation results, and the shaded regions reflect the sensitivity of the simulations to the uncertainty in $c = -8.5 \pm 0.1$ Hz. Quench 1 is the same quench as in Fig. 2. Quenches 2 (purple diamonds) and 3 (green squares) stand for $q_i = 1.47q_c$, $q_f = 1.12q_c$ and $q_i = 1.06q_c$, $q_f = 1.003q_c$ accordingly. For the quench 3 data, the uncertainty of c may lead to crossing over to the FM phase. Inset: the fidelity of the ground state F determined from the residual oscillation of $\xi^2_{S_x}$ after the second quench. The maximum fidelity that can be detected (dashed line) is limited by the detection noise.

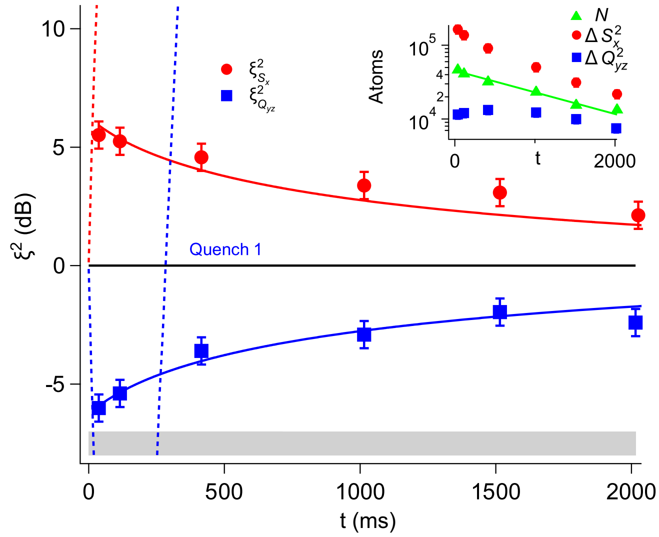

FIG. 4. Measurement of the long-term evolution of $\xi^2_{S_x}$ and $\xi^2_{Q_{yz}}$ in the squeezed ground state. The solid lines are analytical curves based on Eq. (2) with an evolving critical point $q_c(t) = 2|c(t)|$ due to atom loss $c(t) = c(0)\exp(-2t/5\tau)$ [64]. The blue and red dashed lines are the maximum and minimum variance of the deep-quench squeezed state [21]. The inset shows ΔS_x^2 (red circles), ΔQ_{yz}^2 (blue squares), and N (green triangles) versus t.

create the squeezed ground state (see Sec. III in [51]). It is clear that the double-quench method is superior, offering $\geq \sqrt{\eta}$ faster preparation and higher squeezing by minimizing atomic losses.

The degree of squeezing in the ground state increases as q_f approaches q_c according to Eq. (2) because the semiclassical orbits near the pole become more elliptical (Fig. 1). In Fig. 3, noise measurements are made for three different final q_f values to show this dependency. We measure the antisqueezed quadrature $\xi^2_{S_x}$ instead of the squeezing in ΔQ_{yz} to avoid limitations due to the detection noise limit. The sensitivity of the final state on the uncertainty in c (and hence, q_c) increases at higher antisqueezing amplitudes as shown by the shaded envelopes on the simulation curves. Tomographic measurements shown in Sec. III in [51] confirm that the maximum squeezing angle $\theta_s = -\pi$ is independent of q_f.

Following the second quench, any residual oscillation of the measured fluctuations $A = (\max(\xi^2_{S_x}) - \min(\xi^2_{S_x}))/2$ is an indication of imperfect transfer into the ground state. Using a simple harmonic oscillator model [39], and defining $F = |\langle\Psi(t)|\Omega\rangle|^2$ as the fidelity of the targeted ground state $|\Omega\rangle$ of $\hat{H}(q_f)$, the fidelity can be determined from the oscillation amplitude through

$$F \approx 1 - (A/2\eta)^2. \tag{4}$$

Using this result, we determine that $F > 98\%$ for squeezed ground states as shown in the Fig. 3 inset. For quenches to a q_f that approach closer to the critical point, the same small uncertainty in c will lead to a lower fidelity of the final state, as shown in the data.

In Fig. 4, the long-term evolution of the squeezed ground state is measured. The ground state maintains squeezing for over 2 s, and spin-noise tomography shows that the minimum squeezing quadrature angle remains fixed at $\theta_{s,\min} = -\pi$ throughout the entire evolution (see Sec. III in [51]). Interestingly, the gradual decrease in the degree of squeezing is well modeled by a slow tuning of the Hamiltonian due to the loss of atomic density and does not require additional spin decoherence models. Within the Thomas-Fermi model, atom loss due to the finite lifetime of the condensate leads to a decrease in peak density n_0, with $n_0 \propto N^{2/5}$. This in turn affects the spinor dynamical rate and the QCP because $q_c \propto c \propto n_0$ [64] (see Sec. II in [51]). Hence, as the condensate decays, q_f/q_c will increase, leading to a decrease in the squeezing assuming that the state follows adiabatically. The data in Fig. 4 show this trend and compare well with Eq. (2) that include density changes due to atom loss with an exponential time constant $\tau = 1.5$ s. The inset shows directly the time evolution of the variances ΔS_x^2 and ΔQ_{yz}^2 together with the exponentially decaying total atom number, N.

In summary, we have realized spin squeezed ground states in a spin-1 BEC using a double-quench method with a final Hamiltonian tuned close to the quantum-critical point. The squeezing is maintained for over 2 s at a constant orientation angle. The double-quench method can be easily adapted to (pseudo) spin-1/2 systems such as bosonic Josephson junctions [65], which are a special case of the Lipkin-Meshkov-Glick model [66]. It can also be employed for antiferromagnetic spin-1 condensates ($c > 0$) [61,67–70] that have a QCP at $q = 0$ but lack a continuous quantum phase transition.

More generally, we hope that this Letter will have applications to investigations of many-body entangled ground states in the vicinity of quantum-critical points [71]. Those quantum-critical states fluctuate at all length scales and show long-range order, for example, in condensed matter systems [72,73]. We hope this Letter will also facilitate the study of eigenstate entanglement in collective systems [74] and long-term evolution of squeezing state and entanglement.

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Note added.—Recently, a related work was published [75].

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